International Trade, Financial Constraints and Firm Dynamics

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Abstract

This paper investigates the growth trajectory of future multiple product exporters through developing and structurally estimating a model in which firms are heterogeneous in their productivity and assets. Through analytical derivations, I show that when firms are liquidity constrained, the sequence of product introduction depends on firms' initial asset level. In particular, liquidity constrained firms with a high productivity and higher initial assets, first enter the foreign market and then increase their product scope in the domestic market. While other firms, with a similar level of productivity but lower initial assets, accumulate assets through increasing their domestic product scope and then export. The model is then calibrated to the US data in 1995-2000. The theoretical predictions are verified in the estimated model, and it is shown that financing constraints mainly affect the young firms by delaying their export decision. Further, I estimate that removing financing constraints would increase the aggregate productivity level by 1.9%.

Keywords. Multiple-product firms, Heterogeneous firms, International trade, Financial constraints

JEL Classification. F40, L11, L25

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1 Introduction

A few firms account for the majority of international trade in each country, usually with a diverse portfolio and exporting to multiple destinations. This raises questions on the process of growth for these firms. Firms are not born superstars and they are certainly not born exporters, such characteristics grow over time. One possible reason for why certain firms do not reach their optimal size immediately is insufficient access to finance. Evidence suggests that financing constraints exist for both exporting and non-exporting firms.[1] These constraints restrict the set of production options that are feasible for the firm and can explain why large multiple product firms take time to break into different markets.

This paper seeks to understand how multiple product exporters rationalise their production decisions. Are these firms multiple product domestic producers that later start exporting? Or after the initial entry to the domestic market, they start exporting their single product and then expand both into the domestic and international markets? This paper develops a theoretical framework and a structural model and investigates different production paths of firms in becoming diversified exporters.

The framework presented in this paper builds on Melitz (2003) and Bernard et al. (2006) with heterogeneous productivity across firms, and within firms between the varieties they produce. For every new variety the firm adds or exports there are upfront costs to be paid and in order to pay for these costs firms accumulate assets. Exporting is more costly but can generate more profits if the firm has a sufficiently high productivity. Therefore, firms have to decide between two options. They can expand in the domestic market with a new product that has low upfront costs but leads to low profits, or they can enter the foreign market (export) and pay a higher upfront cost but in return earn higher profits. This decision is motivated by firms' financial constraint and thus being prevented from becoming active in all profitable markets immediately.

Liquidity constraints affect the process of growth of future multiple product exporters and delay the entry of firms with lower initial assets to the export market.

[1]See for examples of exporting Muûls (2008) for Belgian firms and Manova (2012) using a large panel of countries. For an example of domestic production see Aghion et al. (2010).

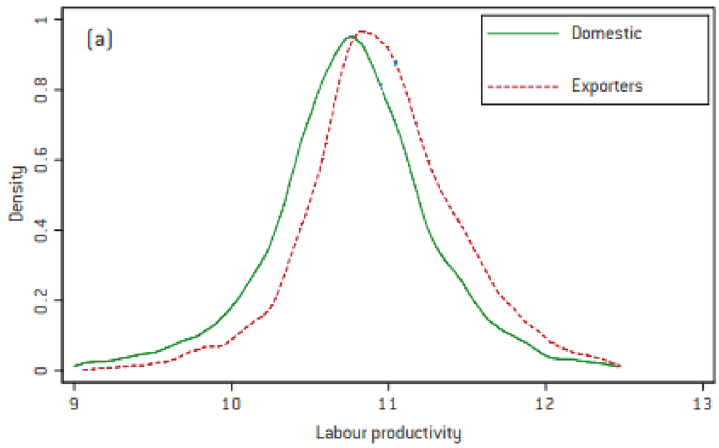

Figure 1: Belgian exporters have a productivity overlap with Belgian domestic producers. Note: data for Belgium for 2004. Source: Mayer and Ottaviano (2008).

These constraints create a trade off for the firm, thus making different orderings of product introduction optimal for firms depending on their initial asset level. Firms with higher initial wealth find it optimal to first export their most productive good then add new varieties, while other firms first build up their assets by increasing their product scope in the domestic market then begin exporting.

Adding financing constraints to a Melitz (2003) type model leads to resource misallocations and inefficient entry and exit. Additionally, liquidity constraints explain why we observe firms with similar levels of productivity behaving differently with regards to exporting. Non-exporters with a high level of productivity would like to export, but are prevented from doing so, as they do not have access to sufficient liquidity to cover their costs. This suggests productivity level alone, is not a sufficient measure of the export status of the firm. The asset level of the firm as a new dimension of heterogeneity, shows different strategies of firms with similar levels of productivity in terms of domestic expansion and exporting capability. Figure 1, from Mayer and Ottaviano (2008) shows the overlap in the productivity level of exporters and domestic producers in Belgium. It is evident from the figure that the productivity level alone is not enough to determine whether a firm is an exporter or not.

The first part of the paper, develops a theoretical model and derives the conditions under which firms first expand in the domestic market rather than exporting. In particular, it is possible to show that there is an asset cutoff for every level of productivity, below which firms will find it optimal to expand their domestic product scope prior to exporting.

The second part of the paper, estimates the model structurally, allowing for uncertainty and entry and exit. In this dynamic setting, the set up constraints the number of products per firm to two. Uncertainty is introduced in the form of shocks to the fixed production costs of the firm,[2] thus making exporter firms subject to additional cost shocks. At every time period, firms decide whether to enter in each domestic and export market subject to the upfront sunk costs. Firms decide to exit the market when continuing in such markets is no longer profitable. With the possibility of expanding in the domestic market, some firms undertake costly investments to increase their domestic product scope, slowly building up assets, in order to be able to export. Simulations of the firms' decision confirm the presence of an ordering of product introduction as a function of the firm's wealth. As expected, liquidity constraints create an overlap in the productivity of the exporting firms and domestic producers.

I then use the baseline calibration, and consider a counterfactual scenario to study the costs of financing constraints. In particular, removing financial constraints increases the aggregate productivity by 2%. There are additional welfare gains associated with the increase in the total number of varieties available to the consumers. The increase in the total number of varieties, is partially due to higher share of firms producing multiple products, and partially due to a rise in the number of exporting firms.

The paper is organised as follows. Section 2 provides a brief review of the literature. Section 3 describes the setup of the model. Section 4 discusses the theoretical implications of the model. Section 5 extends the framework to a dynamic setting and Section 6 provides the counterfactual scenario and welfare analysis. Section 7 concludes.

2 Literature Review

First, the literature emphasises firm heterogeneity and the selection of more productive firms into exporting. Second, there is a large body of literature, mostly empirical but also theoretical, documenting the importance of multi-product firms in international

[2] Similar to Caggese and Cuñat (2013).

trade. Third, the literature explores the lack of access to external finance by firms as a source of misallocation. And finally, there is a growing body of literature on the intersection of international trade and financing constraints.

Recent trade models on heterogeneous firms follow Melitz (2003). By highlighting differences in productivity levels of firms and the role of fixed costs in production, these models show exposure to international trade leads to more productive firms selecting into exporting. This result has also been investigated empirically in many countries: Bernard and Jensen (1999) for the US, Aw et al. (2000) for Taiwan and Korea, Delgado et al. (2002) for Spanish firms to name just a few. This paper adds another dimension of heterogeneity to the above framework and accounts for differences in access to finance by firms.

Investment decisions of firms is highly influenced by their access to finance. Financing constraints can force firms to operate at a suboptimal level, therefore, slowing the growth of the economy. Rajan and Zingales (1996) and Fisman and Love (2007) stress the importance of access to finance for reallocation of resources. Aghion et al. (2010) consider a model in which firms can invest in either short-term or long-term projects. They show that the decision of the firm depends on the severity of the liquidity constraints and tight credit constraints can lead to lower growth and higher volatility in the economy.

This paper contributes to the literature examining the impact of financial constraints on exports. Depending on firm's wealth, response of firms with similar levels of productivity to trade liberalisation differ from each other. To explain this, Chaney (2016) adds liquidity constraints to the standard Melitz (2003) model. He shows, wealthier firms inheriting large levels of assets, are more likely to export. Using a large panel of countries, Manova (2012) confirms that export is lower in sectors more dependent on external finance and in countries where the financial institutions are less developed. Caggese and Cuñat (2013) modify the Melitz (2003) model to introduce a dynamic setting in which firms accumulate assets to overcome their financing constraints. They show credit constraints impact the firm's export decision both directly and indirectly through precautionary savings as firms try to avoid a costly bankruptcy. They find substantial productivity losses from the entry decision of firms. These losses will carry on when the model is extended to a multi-product setting.

Financing frictions can show up in form of a sunk cost which has to be paid upfront. Generally, a firm is willing to pay for these high sunk costs only if it expects the high sunk costs will be compensated with future profits. Empirically, the existence of sunk costs for entry into export has been investigated. Roberts and Tybout (1997) for Colombian manufacturing plants, Campa (2004) for Spanish firms ,Bernard and Jensen (2004) for the US, and Bernard and Wagner (2001) for Germany have found strong evidence consistent with presence of high sunk costs exporting. Combining high sunk costs of export with financial constraints has an important implication. There will be an opportunity cost: Instead of exporting, the firm can invest in other projects. Therefore, future stream of profits should at least equal the sunk cost. If the firm does not expect to get compensated for the sunk cost then entry into exports is not optimal.

While the above literature focuses on single product firms exporting, data shows that multi-product firms dominate the international markets. Empirically, Bernard et al. (2007a) using the US data, Andersson et al. (2008) for Sweden,Muûls (2008) for Belgium, Wales et al. (2018) for the UK and Goldberg et al. (2010) for India emphasise on the importance of multi-product firms. Theoretically, Bernard et al. (2011), Eckel and Neary (2010), and Nocke and Yeaple (2006) develop international trade models in which firms manufacture multiple products. The introduction of product scope for firms, generates new dynamics in the model. In response to trade liberalisation, there will be resource reallocations not only across firms, but also within firms. Interacting these dynamics with credit constraints and assessing the decision of the firm is the main aim of this paper.

Another strand of multi-product firm literature highlights the product mix choice of firms across destinations. Using French data Mayer et al. (2014) investigate how competition and geography shapes a firm's product mix in a given destination. Specifically, firms skew their product mix towards their best performing products and towards destinations with a bigger market.

One important novel element of this paper is looking at sequence of product introduction and time to export. While Albornoz et al. (2012) look at sequential exporting and firm's decision in entering new destinations, this paper instead focuses on the product scope of the firm.

3 The Model

This section presents a framework in which firms are liquidity constrained and can choose to participate in multiple product markets. Firms are heterogeneous in their productivity level and access to liquidity. Firms can add additional product lines and are allowed to export, however, to do so they incur sunk costs and fixed costs associated with the setting up and operation of each product line. As firms are required to pay these costs upfront, they are not able to reach their optimal size immediately, and they are forced to produce suboptimally. To overcome the financing constraints and grow, liquidity constrained firms accumulate wealth through the profits they receive. Then they are able to expand their product scope and export.

In what follows, first the model is discussed in a closed economy setting and then it is extended to an open economy setting to incorporate the export decisions of the firms. Then, the equilibrium definition is provided.

3.1 Closed Economy

3.1.1 Demand

The model follows Melitz (2003) with heterogeneous firms in a monopolistic competition set up. There is a continuum of firms producing differentiated varieties that are demanded by households. These varieties are imperfect substitutes and are indexed by $\omega \in \Omega$. Firms themselves are multiple product producers and the varieties each produces is from the same set Ω. Therefore, the specification does not distinguish between varieties that have been produced across firms and within a firm. Finally, consumers' preferences over these varieties exhibit constant elasticity of substitution with parameter σ.

The overall demand in this economy can be written as:

$$Q = P^{1-\eta} \tag{1}$$

Where η is the industry price elasticity of demand and P is the C.E.S aggregate price index which can be generated by:

$$P = [\int_{\omega \in \Omega} p(\omega)^{1-\sigma}]^{\frac{1}{1-\sigma}} \tag{2}$$

Where $\sigma > 1$ is the elasticity of substitution. Note that since there is no uncertainty in the aggregate the price index will be constant in the equilibrium. The associated quantity with this aggregate price can be written as:

$$Q = [\int_{\omega \in \Omega} q(\omega)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{3}$$

Combining (1) and (3) and using the aggregate price definition from (12), yields the demand for each variety as:

$$q(\omega) = \frac{P^{\sigma-\eta}}{p(\omega)^{\sigma}} \tag{4}$$

3.1.2 Production

The specification for the production technology follows Melitz (2003), Bernard et al. (2006, 2011) and Mayer et al. (2014) with fixed costs and labour as the only input of production. To allow firms to produce multiple but finite number of varieties, the specification of Melitz (2003) is extended to augment over the set of varieties the firm produces. Productivity levels differ across firms and within firms across the varieties that a firm manufactures.

Before the initial entry into the market, firms are uncertain about the level of productivity over different products they can produce. To learn the productivity levels and enter, they pay a sunk cost of entry S_e in units of labour. Upon paying the sunk cost S_e they observe their productivity levels over the range of varieties.

Within a firm, the productivity for each variety is specific to that variety and can be decomposed into two parts. The first component is called "core competency" as in Mayer et al. (2014) and is an indication of firms' efficiency. Alternatively it can be defined as the productivity corresponding to the main variety that the firm produces, where the main variety, is the one with the highest level of productivity. The second component of the productivity, is a parameter $\mathcal{C} \in [0, 1]$ which is product specific within a firm and generates different levels of efficiency across firms' products. This assumption is similar to Mayer et al. (2014), as they assume introducing new products pushes the firm away from its core competency and increases the marginal cost of production.

$$\theta_{ci} = \mathcal{C}^{i-1}\theta_c \qquad i \in N$$

Where θ_{ci} is the productivity of a variety with core competency θ_c and variety-specific productivity $\mathcal{C}^{i-1}$. It can be observed from the variety-specific productivity that as firm introduces more products, its efficiency level declines. Additionally, it is worth noting that the the core competency $\theta_c \in (0, \infty)$ is unknown to the firms as is drawn from a cumulative distribution $G(\theta)$. While $\mathcal{C}$ the relative efficiency across products is the same across firms and known to firms in advance. In the remainder of this paper, when possible I suppress core competency subscripts for convenience. Given the definitions above, the per period cost of production for a firm in terms of labour can be presented as:

$$\ell(\theta) = \sum_{i \in N} J_i [f_p + \frac{q(\theta_i)}{\theta_i}] \tag{5}$$

And:

$$\theta_i = \mathcal{C}^{i-1}\theta \qquad i \in N \tag{6}$$

Where J_i in an indicator variable equal to 1 if the firm is active in a product market and f_p is the fixed production cost.

The specification above shows the augmented nature of the production technology. Total labour employed, depends on the number of varieties a firm produces. As firm increases its product scope, additional fixed cost f_p should be paid for each new variety. Variable costs of production are inversely related to the productivity of each variety. It can be observed, from equation (6), that as firm introduces new varieties, its productivity over each variety decreases and therefore the variable costs increase. This condition ensures that each firm produces a finite number of varieties.

3.1.3 Financial Frictions and Asset Accumulation

With perfect financial markets, firms with a sufficiently high level of productivity can operate in all profitable markets. This implies, that in a single product setting, the solution would be similar to the Melitz (2003) and in multi-product setting to Bernard et al. (2006, 2011). With financial frictions, firms have to pay the sunk and fixed costs of production before the profits are realised. This suggests that if the initial asset holding of the firm is low, it may be prevented from expanding its product scope upon entry. In time, sufficiently productive firms, conditional on surviving, accumulate enough wealth to overcome these constraints.

Firms' initial assets determines the extent to which firms can expand into different domestic or international markets. Therefore, after observing their core competency, firms decide production of which varieties can be profitable and which varieties they actually *afford* to produce. Firms then accumulate assets through profits they receive, in order to pay for future production costs. The accumulation of wealth enables them to enter into new markets that they initially could not afford due to their liquidity constraints. Rate of change of financial wealth is determined as follows:

$$\dot{a}_t = r(a_t - \sum_{i \in N-\{1\}} D_{i,t} S_d) + \sum_{i \in N} J_i \pi_t^D(\theta_i) \tag{7}$$

Where $D_{i,t}$ is an indicator variable taking the value one if the firm decides to introduce domestic variety i in period t and takes the value zero otherwise. Therefore change in asset of the firm at time t is equal to the sum of interest payments on net wealth after the payment of the sunk costs and the stream of profits from the active product lines.

3.1.4 Firm's Decision

After paying the sunk costs of entry S_e and observing the productivity level of each variety, the firm decides which products have a sufficiently high level of productivity to generate positive profits. Then, given its liquidity constraints, the firm decides on the time to introduce each variety, the optimal quantity of each variety and their price. The firm's optimisation problem is:

$$V(\theta_c, a_t) = \sum_{i \in N-\{1\}} [\int_{t_i}^{\infty} e^{-(1-\delta)(t-t_i)} \pi^D(\theta_i) - S_d e^{-(1-\delta)(t_i)}] \tag{8}$$

Subject to:

$$\theta_i = c^{i-1}\theta_c \qquad i \in N \tag{9}$$

$$a_t \geq \sum_{i \in N} J_{i,t} f_p + \sum_{i \in N-\{1\}} D_{i,t} S_d \tag{10}$$

$$J_{i,t} = max\{D_{i,t}\}_0^t \qquad \forall t \tag{11}$$

Where $\delta < 1$ is the exogenous probability of death.

As equation (8) shows, the value function V^D is defined as the net present value of current and future profits. Inequality (10) indicates firms assets should be higher than their upfront costs. The final equation shows once the firm pays the sunk cost of producing a new variety it will continue its production in the subsequent periods and therefore the model does not allow exiting from a product market. Note that the value function is defined such that $D_{i,t} = 1$ if $t \geq t_i$.

Demand for variety i of the firm depends on the variety's price and the aggregate price index P. Standard to the literature, it is assumed that a firm's price for a variety does not affect the aggregate price index, since there is a continuum of firms operating in that market. Therefore, a firm's optimisation problem in each product market leads to the standard result that the price of a variety is a mark-up over the marginal cost:

$$p(\theta_i) = \frac{w}{\theta_i}\frac{\sigma}{\sigma - 1} \tag{12}$$

Additionally, since the liquidity requirement for the firms is on the fixed costs of production and not the variable costs, the financial constraints only impact the number of active lines for a firm but not the quantity produced of each variety. Therefore, the above pricing strategy will hold for all firms and all varieties within a firm.

3.1.5 Firm-product Profitability

Demand for a variety depends on the variety's price relative to the aggregate price P. Given the pricing rule for variety i of the firm defined as equation (12), and normalising the wage $w = 1$, the revenue and the profits of the firm over each variety it produces can be presented as:

$$r(\theta_i) = p(\theta_i)q(\theta_i) = (\frac{w}{\theta_i}\frac{\sigma}{\sigma - 1})^{1-\sigma}P^{\sigma-\eta} = QP^{\sigma}(\theta_i\frac{\sigma}{\sigma - 1})^{\sigma-1} = R(P\theta_i\frac{\sigma}{\sigma - 1})^{\sigma-1} \tag{13}$$

Where R denotes aggregate expenditure. Similarly, the profits can be written as:

$$\pi^D(\theta_i) = \frac{r(\theta_i)}{\sigma} - f_p \tag{14}$$

The profit function presented in equation (14) is similar to the specification in Melitz (2003) which leads to a zero profit productivity cut-off for firms. In the setup of this model the zero profit cut-off is different from the one defined by Melitz (2003), as there are additional sunk costs of setting up a product line. Lemma 1 defines zero net profit cut-off θ^{**}, independent of the initial wealth, such that the firm will enter the product line if it draws a value for θ_i which equal to or greater than θ^{**}.

Lemma 1: *There exists a zero net profit cut-off θ^{**} for each variety $i \in \mathbb{N} - \{1\}$, such that the firm will add a new product if the productivity corresponding to that variety is equal or greater than θ^{**}.*

$$\theta_i^{**} = \left(\frac{S_d \int_0^{\infty} e^{-(1-\delta)t} dt}{f_p} + 1\right)^{\frac{1}{\sigma-1}} \theta^*$$

Where θ^* is the zero profit cut-off productivity as defined by Melitz (2003).
Proof: See appendix.

The sunk cost S_d raises the entry productivity cut-off above θ^*, since now not only the revenues have to cover the fixed cost of production, but the stream of revenues should compensate the firm for the initial sunk cost paid. The two cut-offs coincide if $S_d = 0$.

3.2 Open Economy

In this section, I will extend the closed economy framework described in the previous section to an open economy setting and assume all trading partners are symmetric in terms of preferences and production technology. I denote export market variables with a subscript x and domestic market variables with subscript d. In line with the empirical literature, I assume that participating in international trade is costly. There are additional sunk costs, fixed costs and variable costs associated with exporting. The sunk costs of entry into the export market can be interpreted as the cost of initial research required before starting to export. There is a large body of empirical evidence supporting the existence of such high sunk costs for exporting. It is assumed that the sunk cost is incurred for each product that the firm decides to export and is denoted by S_x. Further, there are product specific fixed costs, f_x representing product specific costs such as the cost of advertisement. Finally, there are iceberg costs of $\tau > 1$, showing the shipments costs, as it is standard in the literature.

3.2.1 Production and Firm-Product Profitability

The specification follows Melitz (2003) and as before is augmented to allow for production and export of multiple goods. Since the countries are symmetric, firms face the same elasticity of demand in all countries and export prices are a constant multiple of the domestic prices:

$$p_x(\theta_i) = \tau p_d(\theta_i) \tag{15}$$

As described above, if a firm exports a variety it has to pay additional fixed cost f_x for each product that it exports. In addition, total quantity produced for varieties that are exported will increase. $q(\theta_i)$ therefore, includes both the quantity supplied to the domestic market $q_d(\theta_i)$ and the quantity exported $q_x(\theta_i)$. Hence, the total labour employed increases for exporter firms by the amount of fixed costs f_x and variable costs $q_x(\theta_i)/\theta_i$, which is implicitly included as an increase in $q(\theta)$. The production technology now can be written as:

$$\ell(\theta) = \sum_{i \in N} J_i [f_p + \sum_{J_i^X \in J_i} J_i^X f_x + \frac{q(\theta_i)}{\theta_i}] \tag{16}$$

Where:

$$\theta_i = \mathcal{C}^{i-1}\theta_c \qquad i \in N \tag{17}$$

Similar to the closed economy section, J_i is an indicator variable taking the value 1 if the firm is active in production of variety i. In a similar vein, J_i^X is an indicator variable taking the value 1 if the firm exports variety i.

Additional fixed and sunk costs of exporting imply that it is optimal for a firm to serve the domestic market prior to exporting. Given the characterisation of the production technology (the cost function), therefore, the revenues and costs of production can be proportionally divided between the domestic market and the export market. Therefore, a firm deciding whether to export or not, compares the fixed and sunk cost of exporting with its respective revenues only. Given the pricing rule for exports, the firm's revenue can be written as:

$$r_x(\theta_i) = \tau^{1-\sigma} R (P\theta_i \frac{\sigma}{\sigma - 1})^{\sigma - 1} \tag{18}$$

The above equation suggests that similar to the domestic market, there exists a productivity cut-off below which the profits generated from exporting a variety are negative and it is not optimal for the firm to export that variety.

Lemma 2: *There exists a zero net profit cut-off for exporting* θ_x^* *for each variety* $i \in \mathbb{N}$ *such that the firm will export the product only if the productivity corresponding to that variety is equal or greater than* θ_x^*:

$$\theta_x^* = (\frac{S_x \int_0^\infty e^{-(1-\delta)t} dt}{f_x} + 1)^{\frac{1}{\sigma-1}} \theta_{x,m}^*$$

Where $\theta_{x,m}^*$ *is the Melitz (2003) exporting cut-off productivity.*
Proof: See appendix.

It can be observed that θ_x^* is strictly greater than $\theta_{x,m}^*$ as long as $S_x > 0$. This is expected as additional sunk costs imposed on firms should be compensated by the stream of profits in later time periods. This cut-off is independent of the initial level of wealth and provides the lower bound for entry of firms into exporting.

Since all goods are identical in terms of fixed and sunk costs, the zero net profit cut-off for exporting is the same across all the products the firm manufactures. The intuition underlying the relationship in the export market is similar to the one discussed for the domestic market. An increase in the sunk costs of exporting raises the net cut-off above the exporting cut-off as defined by Melitz (2003). Finally, similar to results in Melitz (2003), opening up to trade, increases the zero profit entry cut-off through a decrease in aggregate price level P.

3.2.2 The Firm's Decision

Analogous to the closed economy case, the firms decide on the time to add each variety in the domestic market t_i, prices p_i and quantities q_i. Additionally, with opening up to trade, the firms decide on the set of products to export and the time to export them t_i^X. The decision to export is denoted by $X_{i,t}$ which is a binary variable taking the value 1 if the firm decides to export variety i at time t.

Define V^X as the value of a firm that can operate both in home and international markets. Then a liquidity constrained firm decides at any t between expanding in the domestic market or exporting the most productive product that is not already exported. $D_{m_t} = \max\{D_{i,t} = 1\}_{i=1}^{N}$ shows the final variety added to the domestic market by time t. Similarly, $X_{m'_t} = \max\{X_{m'_t+1,t} = 1\}_{i=1}^{N}$ shows the final variety exported by time t. Given the above definitions, $X_{i=m'+1,t}$ is equal to one (i.e., the firm begins exporting a new variety in period t) when the following conditions are satisfied for all t and all $m' \leq m$:

$$V^X(\theta_c, a_t)_{|X_{m'+1,t}=1} > V^X(\theta_c, a_t)_{|D_{m+1,t}=1} \tag{19}$$

$$V^X(\theta_c, a_t)_{|X_{m'+1,t}=1} \geq V^X(\theta_c, a_t)_{|X_{m'+1,t}=0} \tag{20}$$

The conditions are written for a liquidity constrained firm under the assumption that the decision to expand or export in not reversible in the subsequent periods. However, the firm can always decide to exit the market entirely if it is not profitable as a whole. This assumption is justified since exiting the export market is not optimal when sunk costs of exporting are large. Condition (19) states that the value of the firm exporting variety $m' + 1$ (i.e., the variety with the highest level of productivity that is not already exported) must be greater than the value of adding a new variety in the domestic market. Condition (20) states that the value of exporting the new variety must be greater than the value of not exporting it. Similarly, these conditions can be written for a liquidity constrained firm that decides introducing a new variety in the domestic market is optimal. $D_{m+1,t}$ is equal to one if:

$$V^X(\theta_c, a_t)_{|D_{m+1,t}=1} \geq V^X(\theta_c, a_t)_{|X_{m'+1,t}=1} \tag{21}$$

$$V^X(\theta_c, a_t)_{|D_{m+1,t}=1} \geq V^X(\theta_c, a_t)_{|D_{m+1,t}=0} \tag{22}$$

The interpretation of the above conditions is similar to (19) and (20). Condition (21) states that adding a new variety in the domestic market must put the firm on a higher value path compared to exporting an already existing variety. The next condition indicates producing a new variety must generate higher net profits than not producing the variety. Now, $V^X(\theta_c, a_t)$ can be determined as the net present value of future profits. The value of the firm can be written as:

$$V^X(\theta_c, a_t) = \sum_{i \in N-\{1\}} [\int_{t_i}^{\infty} e^{-(1-\delta)(t-t_i)} \pi^D(\theta_i) - S_d e^{-(1-\delta)(t_i)} \\ \int_{t_i^X}^{\infty} e^{-(1-\delta)(t-t_i^X)} \pi^X(\theta_i) - S_x e^{-(1-\delta)(t_i^X)}] \tag{23}$$

Subject to:

Conditions (19), (20), (21), (22) and

$$J_{i,t} = max\{D_{i,t}\}_0^t \tag{24}$$

$$J_{i,t}^X = max\{X_{i,t}\}_0^t \tag{25}$$

$$\theta_i = \mathcal{C}^{i-1}\theta_c \qquad i \in N \tag{26}$$

$$a_t \geq \sum_i J_{i,t} f_p + \sum_i D_{i,t} S_d + \sum_i J_{i,t}^X f_x + \sum_i X_{i,t} S_x \tag{27}$$

$$\dot{a}_t = r(a_t - \sum_i D_{i,t} S_d - \sum_i X_{i,t} S_x) + \sum_i J_i \pi_t^D(\theta_i) + \sum_i J_{i,t}^X \pi^X(\theta_i) \tag{28}$$

Where the value function is the present discounted value of the future profits both from domestic production and exporting. S_d and S_x denote the sunk costs of adding domestic production lines and exporting new products respectively. As before $e^{-(1-\delta)t}$ is the discount factor, where δ is the exogenous probability of death. $J_{i,t}^X$ is an indicator variable, taking the value one for a firm exporting variety i. If a firm pays the sunk cost of exporting at any arbitrary time $t' < t_i^X$ then $X_{i,t'} = 1$ and the firm takes the exporter status for variety i. The inequality (27) shows that at any point in time the firm should have enough wealth to pay for the upfront costs of production and exporting otherwise it has to exit the market. Condition (28) is the asset development equation.

There is a substantial difference between the firm's decision in a closed economy setting and open economy setting. In a closed economy, the initial asset level of the firm determines the number of products the firm produces in the first time period. Then, as firm accumulates assets it starts adding new varieties as long as they generate positive net profits. The order of adding these new varieties is solely based on the productivity and independent of wealth.

In an open economy, the firm faces a trade off which was not present in a closed economy setting. Now the firm has to decide between adding another line in the domestic market or exporting its most productive variety, given it has a sufficiently high productivity to do both. This suggests that the optimal decision of multiple-product firms with financial constraints, includes a sequence for introducing new products and depends not only on the productivity level but also the assets of the firm. Conditions (19)-(22) summarise the decision of the firm and the sequence of product introduction will be further explored in section 4.1.

3.2.3 Entry Decision

Firms have to pay a sunk cost to observe their core productivity level θ_c. After observing the productivity level, firms decide on the varieties they produce and the markets they serve conditional on having sufficient funds to pay for the sunk costs of entering these markets. Free entry requires, ex-ante, the expected value of the entry and learning the core competency be equal to the cost of entry S_e:

$$E[V^X(\theta_c, a_0)] = S_e \tag{29}$$

Where the operator E refers to expectation over core competency θ_c. In models without liquidity constraints, the free entry condition provides a unique productivity cut-off above which firms enter in the equilibrium. With financing constraints, however, the expected net present value of profits is a function of firm's assets. The initial asset level of the firm affects the expected time period in which firms can pay for the sunk costs of entering into the other markets, conditional on having a sufficiently high productivity. This means, the value of observing a certain level of productivity is different among firms with different levels of initial assets. A firm with high initial wealth, immediately pays for the costs of entry into all profitable markets. With a lower level of wealth, the firm cannot afford to become active in all the profitable markets right away. Therefore, he delays the decision to enter other markets and instead accumulate assets. Hence, the net present value of profits is an increasing function of the initial asset level. The characterisation for the entry condition is provided in section 4.2.

3.3 Equilibrium

The steady state equilibrium is characterised by an aggregate price P, an aggregate quantity Q, and time invariant distributions of operating and entrant firms over their productivity levels and asset level such that firms maximise their value functions $V^X(\theta_c, a_t)$ defined in (23) given conditions (24)-(28). Existing firms decide to expand in the domestic market only if conditions (21) and (22) are satisfied. Existing firms decide to export each product according to (19) and (20). New entrants satisfy condition (29).

The mass and distribution of firms over the productivity levels determines the distribution of prices. The CES aggregator then determines the aggregate price level P. The presence of the exogenous exit probability δ ensures that the distribution of wealth across firms does not grow without bounds.

4 Theoretical Implications

4.1 Product Sequencing

Section 2.2.2 briefly discussed the choices financially constrained firms face regarding introducing new products. This section explores the decision of the firm in more details and characterises the firms' decision as a function of initial wealth. Before moving on to derive this condition, I explain the effect of the asset level and the option to produce multiple varieties on firms' production strategies.

First, to isolate the effect of financial constraints, I abstract from a multiple product setting and consider the case in which firms can only manufacture a single product. Financial constraints, as before, come into effect through upfront costs associated with production and exporting. Therefore, the initial asset level of the firm generates differences in export behaviour of the firm. All firms with high enough level of productivity profit from exporting. Unconstrained firms have access to the required liquidity to pay the upfront costs and start exporting. However, the upfront costs hinder firms with lower level of assets from entering the export market. These firms are initially only active in the domestic market. They start exporting at a later point and only when they accumulate sufficient assets to pay for the upfront costs. This provides evidence on results observed in Figure 1 showing that firms with similar levels of productivity sometimes export and sometimes not, and justifies the inclusion of wealth as another dimension of heterogeneity.

Enriching the space of products will allow for other mechanisms that further reinforce differences in the behaviour of the firm. Specifically, in a multiple product setting, firms with limited initial assets have to choose between investing in domestic expansion *and* exporting. This means, for firms the decision to invest in domestic expansion comes at the expense of delaying the export of the existing goods. The strategy of the firm depends on the discounted present value of taking each path. The present value of each path is a function of the productivity level of the firm for each product, its asset level and the sunk costs associated with domestic production and exporting.

To show how the asset level of the firm affects its production strategies, I consider an n good economy. Denote $\{\theta_1,\ \theta_2,\ ...,\ \theta_n\}$ the productivity set for all products of a given firm relating to varieties $\{1, 2, ..., n\}$ respectively. The main result states that there exists a threshold asset level, such that a firm with an initial asset below the threshold, follows a path of expanding in the domestic market prior to exporting. On the other hand, above this threshold, it is optimal for firms to first export then add the second variety in the domestic market. Therefore, the sequence of product introduction changes depending on the firm being financially constrained or not: firms with a lower access to liquidity add varieties in the domestic market while firms with higher asset levels become unconstrained through exporting. The proposition below characterises the firm's strategy.

Proposition 1 *There exists strictly positive productivity levels θ_x^* and a finite productivity level θ_{high}such that:*

1. *For firms with productivity level $\theta_c \leq \theta_x$ it is optimal to only produce for the domestic market.*

2. *For firms with productivity level $\theta_c \in [\theta_x^*, \theta_{high}]$ it is always optimal to expand in the domestic market before starting to export.*

3. *For firms with productivity level $\theta_c \geq \theta_{high}$, there exists an initial asset level $a_{t-1}(\theta_c, \theta_2, ..., \theta_n)$ such that below this level firms will first expand into the domestic market and above which they will overcome their credit constraints through saving and exporting their most productive good prior to increasing their product scope in the domestic market.*

Proof: See appendix.

In particular, given the sunk costs and fixed costs of production, the initial asset level of the firm as a function of its productivity level determines the present value of each path and therefore the sequence of products introduction. This means differences in asset level of firms with similar levels of productivity will make different paths optimal for firms. Some will find expanding in the domestic market prior to exporting optimal while others choose to first export then add new varieties in the domestic market. Note that if the firms survive then those with similar levels of productivity end up having the exact same product mix. It is only the sequence of introduction of these goods that differ between them. However, with the exogenous probability of death a fraction of these firms do not live long enough to become unconstrained and active in all the markets that is profitable for them.

4.2 Free entry condition- 2 products

In this section, I show that the productivity cut-off of entry for firms depends on their initial asset level. The characterisation of the free entry condition for a two product world with financially constrained firms is then provided in the appendix.

Intuitively, financing constraints affect the time at which the firm is able to enter new markets. This delay in accessing new markets, decreases the value of entry for the firm. To compensate for this decrease in the value, the firm needs to observe a higher level of productivity to break even. A higher productivity cut-off, increases profits at each time period and the value of the firm such that the free entry condition will hold.

Proposition 2 (Value of entry) *In the presence of liquidity constraints and upfront costs of production, the value of entry is a continuous and weakly increasing function of the initial asset level a_0.*

Proof: See appendix.

In a single product setting, financing constraints delay the entry into the export market. This delay is required for the firm to accumulate assets for paying the upfront costs. Extending the setup to allow firms to manufacture and export two products generates additional dynamics. The effect of financial constraints, as before, is delaying entry into different markets. In particular, in a two product setting, there are four different markets a firm can *potentially* serve: variety 1 in the domestic market, variety 2 in the domestic market, variety 1 in the international market and variety 2 in the international market. The option to produce multiple products, also adds another dimension to the decision making by the firm. As discussed in the previous section, multiplicity of markets, imply that a liquidity constrained firm, now needs to decide on the ordering of entry into these markets. Firms optimally enter any market, whether domestic or international, with the variety that generates the highest level of profit. Since the marginal cost of production of each variety is inversely related to its level of productivity, firms start production or exporting with variety 1. Additionally, the selection condition to the export market is imposed.[3] This ordering has been characterised discussed in length in the previous section.

5 The Dynamic Model

In this section the model is extended to a dynamic setting to be solved numerically. The framework remains as described in the previous section, but a simpler case in which firms can produce and export two varieties is considered. In a two product setting, liquidity constrained firms have to decide between introducing variety two to the domestic market or exporting variety one. Therefore, the main result of the theoretical model, sequence of product introduction with regard to initial wealth, can be illustrated in this setting.

In this setting, the firm's profit is subject to shock ϵ that follows a two state symmetric Markov process in which the firm will receive either a positive or negative shock to its costs. The probability of remaining in the same state is equal to ρ and

[3] see Melitz (2003)

the probability of changing the state is $1-\rho$. After observing the sunk cost of entry S_e the firm is assigned with the initial shock with equal probability. Therefore, the initial distribution over the value of the shock is uniform. The firm also observes the core competency level which is drawn from an exponential distribution for the entrants and is specified as in Caggese and Cuñat (2013). I assume all entrants are financially constrained however the extent to which they have access to liquidity differs between them.

To model is solved numerically and in discrete time. The discrete time setup and the numerical solution to the model are described in the appendix.[4]

5.1 Calibration

This section calibrates the model to match the moments of data for the sample of US exporter firms between 1995 -2000. Table 1 provides the exogenously calibrated variables, their values and their source.

Table 1: Parameters- Externally Calibrated

	Parameter	Value	Source
Interest rate	r	0.04	US interest rate year 2000
Elasticity of substitution	σ	4	Costantini and Melitz (2008)
Industry elasticity of substitution	η	1.5	Costantini and Melitz (2008)
Iceberg costs	τ	1.2	Costantini and Melitz (2008)
Death shock	δ	0.15	Costantini and Melitz (2008)
Cost shock correlation	ρ	0.7	Caggese and Cuñat (2013)

It is assumed that upon paying the sunk cost of entry S_e entrants draw their core competency from an exponential distribution with mean λ truncated across the productivity (core competency) space. Entrants are all financially constrained and are uniformly distributed across the lower levels of asset grid, such that all entrants remain financially constrained. Similarly the initial value of the cost shock is randomly drawn from a uniform distribution. I assume that for any extra product the firm adds to its portfolio, the productivity of the new product moves one step down on the productivity grid. Furthermore, exporters are subject to additional cost shocks which

[4]While setting up the model in discrete time is advantageous, the theoretical results as described in section 4 will not be as strong. This has been discussed in details in the appendix.

similar to before follow a two state Markov process indicating that exporting exposes firms to conditions of other markets. The domestic and export shock are assumed to be independent of each other. The appendix provides more details.

Table 2: Parameters- Internally Calibrated

	Parameter	Value
Sunk cost of entry	S_e	.60
Sunk cost of adding a domestic line	S_d	.02
Sunk cost of export	S_x	.98
Fixed cost of domestic production	f_p	.03
Fixed cost of exporting	f_x	.09
Parameter of entry distribution	λ	1.10

The model is calibrated such that the percentage of exporting firms and multiple product exporters match the empirical moments. To fit the model with empirical data, sunk costs, fixed costs and the demand parameter are simultaneously chosen. Sunk cost and fixed cost of exporting jointly determine the share of exporters and multiple product exporters. Sunk cost of domestic production pins down the share of multiple product firms in the economy and fixed cost of domestic production together with fixed cost of exporting match share of fixed cost in the economy. Sunk cost of entry and the parameter of entry distribution affect the distribution of incumbents by affecting the entry cutoff and the shape of entrants' distribution.

Table 3: Moments

	Data	Simulated moment	Source
Share of multi product firms	.40	.56	Bernard et al. (2010)
Share of exporters	.15	.19	Bernard et al. (2011)
Share of multiple product exporters	.58	.58	Bernard et al. (2011)
Share of fixed costs	.20	.17	Costantini and Melitz (2008)
Output share of single product firms	.13	.16	Bernard et al. (2007b)
Shipment value of exporter to non-exporter	6.4	7.1	Bernard et al. (2007b)

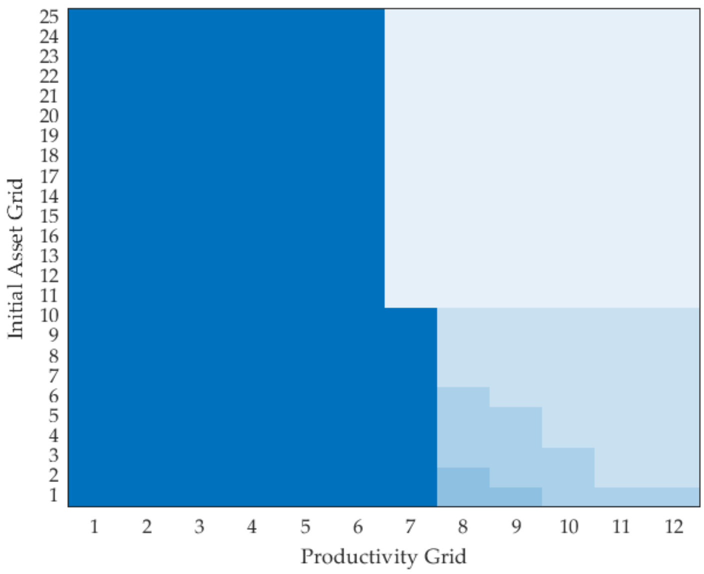

Figure 2: Exporting time as a function of initial assets and productivity. Darker colours show longer delays, and lighter colours are indicative of low waiting time to export.

5.2 Results

5.2.1 Exporting, Productivity and Initial Assets

In the steady state distribution of firms, there is an overlap on the productivity level of exporters and non-exporters suggesting that productivity level is not enough to determine the status of a firm as an exporter. In this regard, Figure 2 describes the cutoff of entry into exporting as a function of firm's initial asset, as well as the expected waiting time for a given firm to be able to start exporting. The very dark blue captures firms that do not export, and thus present the cutoff for exporting as a function of state variables. It can be observe that productivity level alone is not enough in describing the export status.

Firms on the top right hand side of the heat map are high productivity firms with high assets, and can export immediately. The bottom right section of the heat map, has more heterogeneity and points out to firms that are building up their assets to be able to pay for the costs of exporting. Lack of access of these firms to sources of finance creates resource misallocation and leads to lower aggregate productivity. The figure also shows that firms higher on the productivity grid outgrow their financing

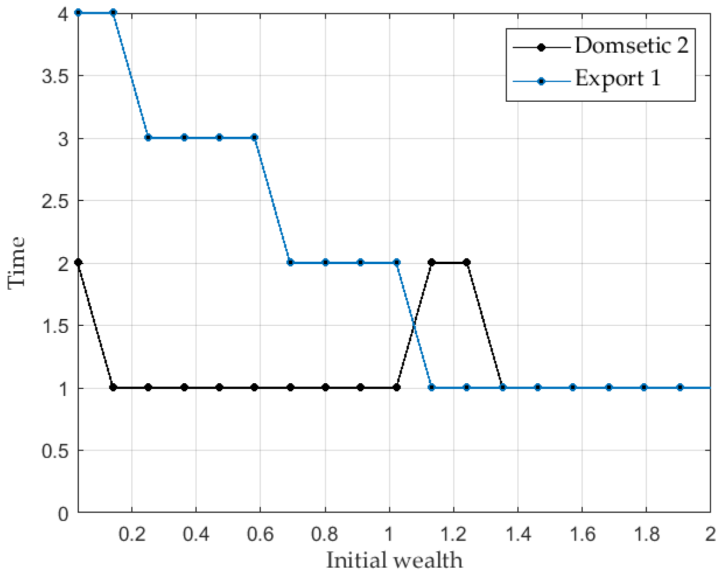

Figure 3: The figure shows the decision of the firm regarding the time to export the existing product or add a new product in the domestic market as a function of the initial wealth.

constraints relatively faster, with lighter colours presenting lower waiting times.

5.2.2 Simulations of the Proposition

Figure 3 shows the implied dynamics by the Proposition 1 at firm level in a two-product world. It illustrates the ordering decisions of the firm regarding export of the existing variety and expanding in the domestic market with a new variety given the initial asset level. The first and second statements of Proposition 1 show the cut-off for entry into the export market and do not have implications for sequence of product introduction. Therefore, I focus on Statement 3 of the proposition.

Figure3 shows the decision regarding the time to expand or export depends on firms' initial asset level. For very low initial asset levels, the firm does not have access to enough liquidity to enter either of these markets immediately. For example take value 0.03 for initial wealth. The time to introduce variety 2 to the domestic market is $t = 2$ while the time to export variety 1 is $t^x = 4$. While there is a delay in entry to both markets (i.e. the firm cannot enter at time $t = 1$), the time to expand in the domestic market comes before exporting the existing variety 1 for this asset level.

Now, take the value $a = 1.2$. For this value, the firm exports at $t^x = 1$ while the

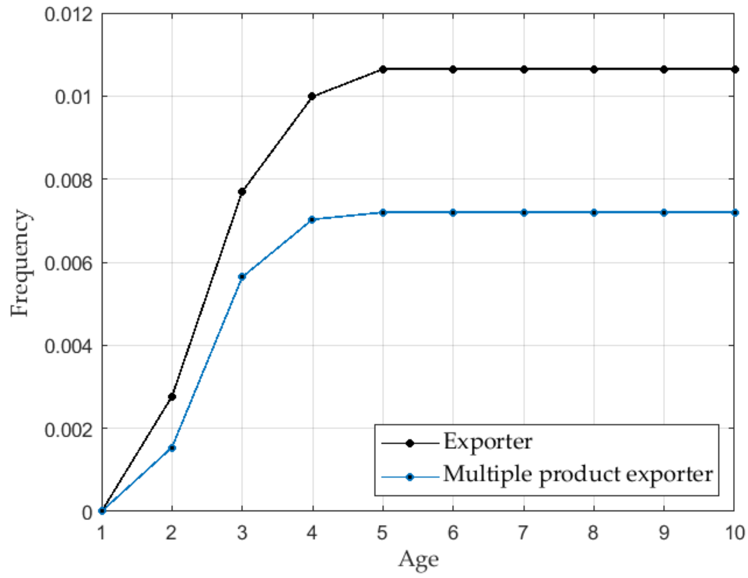

Figure 4: Average frequency of exporting firms as a function of age

decision to add a new domestic line comes at $t = 2$. Therefore, it can be seen that depending on the initial value of wealth the ordering of product introduction changes. The figure is therefore able to capture the non-monotonicity implied in the policy function of the firm. Note that, exporting product 2 always comes at the end and therefore is not included in the figure.[5] Finally, for very high initial levels of wealth the firm is able to enter all the markets immediately.

5.2.3 Firm Dynamics

Figure 4 and Figure 5 show the implied dynamics at the firm level. Figure 4 plots the probability of being an exporter as a function of age. Similarly, the average frequency of multiple product exporters is shown in the same figure. Young firms, on average have a lower probability of being an exporter, as they are liquidity constrained. However, by the age of five, firms will outgrow their financing constraints.

Figure 5 shows the development of financial assets of firms. Exporters accumulate more wealth since they are more productive in general and they serve multiple markets. It can also be observed that the rate of increase in assets of exporters increases with

[5]Since the simulations are done in discrete time, there will be some additional dynamics which were not present in a continuous time framework. This has been included in the appendix.

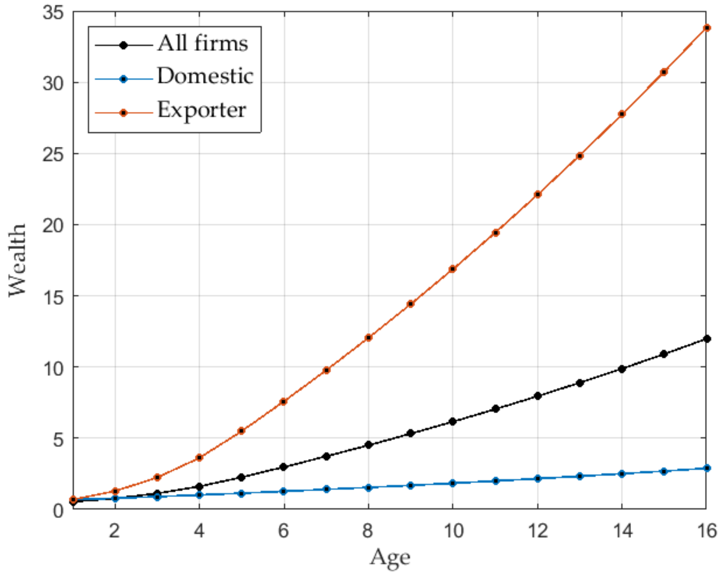

Figure 5: Average firm asset as a function of age

age for younger firms. This can be explained by lack of access to liquidity. A fraction of young exporters are liquidity constrained and therefore cannot access all the markets that generate profits for them. Through time, these firms outgrow their constraints and their wealth increases at a higher rate.

6 Counterfactual Scenario and Welfare Analysis

In this section, a counterfactual scenario in which firms do not face financing constraints is considered. The model is analysed under parameter values reported in Table 1 and Table 2. The aim of this exercise is to understand the inefficiencies and welfare losses due to financing frictions. Table 4 reports the results.

Table 4: Moments - Counterfactual

	No financing constraint	Financing constraint
Share of multi product firms	.78	.56
Share of exporters	.36	.19
Share of multiple product exporters	.52	.58
Share of fixed costs	.20	.17
Output share of single product firms	.02	.16
Shipment value of exporter to non-exporter	31.3	7.2

Table 4 shows that in response to removing financing constraints, the share of multiple product firms and the share of exporters will increase. Share of multiple product exporters falls, due to having a higher share of single product exporters (in levels both values increase). The number of single product exporters increases, as firms no longer have to rationalise the sequence of product introduction and can immediately enter all profitable markets. As expected, output share of single product firms falls, as now more firms are able to produce multiple products. Further, shipment value of exporters increases with respect to non-exporters as access to financing allows shipping to additional markets.

To understand the welfare implications of relaxing financing constraints, I consider welfare per worker measured as $\frac{w}{P}$, wage over aggregate price. As wage is normalised to 1, a decline in aggregate price level leads to welfare improvements.

Relaxing financing constraints can improve welfare through two channels. First, in the absence of liquidity constraints, firms are able to enter all profitable markets immediately, and therefore, there will be more varieties available in total to the consumers. Second, through selection into exporting, similar to the main channel discussed in Melitz (2003), the productivity cut-off of entry increases as now more firms can afford to export. This leads to exit of less efficient firms and improving aggregate productivity level.

$$\tilde{\phi} = \frac{1}{M_t}\left[M_c\tilde{\phi_c}^{\sigma-1} + nM_{c,x}\tau^{-1}\tilde{\phi_{c,x}}^{\sigma-1} + \sum_i (M_i\tilde{\phi_i}^{\sigma-1} + nM_{i,x}\tau^{-1}\tilde{\phi_{i,x}}^{\sigma-1})\right]$$

Where $\tilde{\phi}$ is the average productivity in the economy. M shows the number of firms, with M_t referring to the total number of firms, M_c number of firms producing their core product in the domestic market, $M_{c,x}$ number of firms exporting their core product and M_i number of firms having their i-th product line. $\tilde{\phi_c}$ and $\tilde{\phi_i}$ refer to average productivity of the core product among all firms and average productivity of the i-th product line among the firms. x subscript refers to exporting of each product and τ is iceberg cost. n is the number of countries, which for the calibration is set out to 5 to match the data counterparts provided in the literature. Given the definition in equation above, the aggregate price level can be presented as:

$$P = M_t^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1}\frac{1}{\tilde{\phi}}$$

The analysis show that welfare improves by 12.2% as a result of removing financing constraints. This is partially due to productivity improvements, where average productivity increases by 1.9%, but the majority of the welfare improvement is due to the number of available varieties which increase by 45%. The variety effect is sensitive to the number of trading partners, that in this exercise is set to five. Adjusting the number of trading partners to 1, decreases the variety effect from 45% to 26%.

7 Conclusion

I present a theoretical model in which firms can produce and export multiple products and they face liquidity constraints. Firms accumulate wealth to overcome their financing constraints that affect the firms' ability to pay for their fixed operational costs and the one-off sunk costs of entry into any new market. In the process of overcoming their constraints, firms face a choice between increasing the number of varieties they produce in the domestic market and exporting the varieties they already have. Exporting is more costly but it can generate higher profits if the firm has a high enough productivity. Therefore, constrained firms face a trade off between increasing their product scope in the domestic market and exporting. The model shows that firms with higher initial assets break into export market faster provided they have a sufficiently high level of productivity. This induces a different sequence of product introduction by firms with different level of initial wealth that are otherwise similar.

The model also shows that in equilibrium, financing frictions lead to misallocation of resources. This occurs because, the lack of entry from firms with lower initial assets is compensated by an increased entry of firms with higher initial wealth but low levels of productivity. As a consequence, there will be substantial welfare losses if a significant number of firms face financial constraints.

Finally, the theoretical framework was applied to a dynamic setting in which firms can produce and export two products. In this setting, then, the theoretical predictions were tested. After the initial entry to the domestic market, firms with high initial asset levels overcome their financing constraints by first exporting the most productive good and then increasing their product scope. While, firms with lower initial wealth accumulate assets slowly by introducing a new variety in the domestic market and then starting to export. Additionally, it was shown that for younger firms, liquidity constraints decrease the probability of exporting while this is not the case for older firms. Also, the average asset level of an exporter is much higher than the average asset of a non-exporter. The difference is more significant as the age of the firm increases.

The paper then studies a counterfactual scenario, in which financing constraints are removed. This leads to an increase in the share of firms producing multiple products and the share of exporter firms. Output share of single product firms falls significantly, as under this scenario, only the least productive firms produce a single product. Further, removing financing constraints lead to 1.9% increase in aggregate productivity and significant efficiency gains.

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Appendix A: Theoretical Proofs

A.1. Proof of Lemma 1

To derive the zero net profit cut-off, first define the zero-profit cut-off θ^*, the productivity level at which the per period profits of the firm are equal to zero. Then:

$$r(\theta^*) = \sigma f_p \qquad \forall \theta_i \tag{30}$$

This characterisation is as in Melitz (2003), and simplifies the revenue of any firm from a given variety associated with productivity level θ to:

$$r(\theta) = (\frac{\theta}{\theta^*})^{\sigma-1} \sigma f_p \tag{31}$$

And profits can be written as:

$$\pi^D(\theta) = ((\frac{\theta}{\theta^*})^{\sigma-1} - 1) f_p \tag{32}$$

For any additional variety the firm adds in the domestic market, there is a one-off sunk cost S_d to be paid. Hence, a firm with the zero-profit cut-off productivity level for one of its varieties, generates negative net profits. The presence of this sunk cost implies a zero net profit cut-off productivity level above θ^* that makes the firm indifferent between adding a new product line and not entering that market. To derive this cut-off, define θ^{**} as the productivity cut-off above which net present value of profits from variety i are positive and define $\Delta = \int_0^\infty e^{(1-\delta)t} dt$ to get:

$$\Pi^D = \int_0^\infty \pi_i e^{-(1-\delta)t} dt - S_d$$

$$((\frac{\theta}{\theta^*})^{\sigma-1} - 1) \frac{f}{\Delta} - S_d = 0$$

Above holds at $\theta = \theta^{**}$.

Therefore:

$$\theta^{**} = (\frac{S_d \Delta}{f} + 1)^{\frac{1}{\sigma-1}} \theta^*$$

Note that if $\theta^{**} \leq \theta^*(a_0)$ then the entry cut-off for new varieties will coincide with $\theta^*(a_0)$. Where $\theta^*(a_0)$ is the value of entry for liquidity constrained firms.

A.2. Proof of Lemma 2

To derive the zero net profit cut-off for exporting, first define the zero-profit cut-off $\theta^*_{x,m}$, the productivity level at which the profits of the firm are equal to zero at a given point in time. This is exactly the same as the productivity cut-off defined in the Melitz (2003) model and leads to Melitz (2003) exporting cut-off:

$$r_x(\theta^*_{x,m}) = \sigma f_x \tag{33}$$

$$\theta^*_{x,m} = \tau[(\frac{f_x}{f_p})]^{\frac{1}{\sigma-1}}\theta^* \tag{34}$$

The set up presented in this paper, however has additional sunk costs associated with exporting. Therefore, the actual cut-off above which firms start exporting has a different characterisation from Melitz (2003). In the presence of the sunk cost S_x, the productivity cut-off is presented by the level at which net present value of exporting is zero. Since $S_x > 0$, it must be that θ^*_x is higher than the$\theta_{x,m}$. θ_x can be written as a function of the Melitz export cut-off $\theta_{x,m}$.

Consider the zero net profit condition for exporting and as before and define $\Delta = \int_0^\infty e^{(1-\delta)t}dt$ to get:

$$((\frac{\theta}{\theta^*_{x,m}})^{\sigma-1} - 1)\frac{f_x}{\Delta} - S_x = 0$$

Above holds at $\theta = \theta^*_x$.

Therefore:

$$\theta^*_x = (\frac{S_x\Delta}{f_x} + 1)^{\frac{1}{\sigma-1}}\theta^*_{x,m}$$

A.3. Proof of Proposition 1

To prove the proposition, I start from a two good case and then generalise.

First, suppose that the firm is constrained with initial asset level $a_0 < a_u$ where a_u shows the asset level for which the firm will be unconstrained. Also suppose that the economy is at its steady state such that the aggregate price is known to the firms.

As before, we continue to rank the products based on their productivity, such that the first variety that the firm produces is the most productive one. The productivity of the second variety is a fraction of the first and so on. Therefore, we have:

$$\pi_1^D > \pi_2^D > ... > \pi_n^D$$

Where π_i^D shows the profit from the domestic market for good i.

To prove the proposition, I start from a two good case and then generalise.

In a two good world, if the firm is liquidity constrained it means it cannot expand to all the domestic and export markets that is profitable for him.

Define:

$$\pi_1^D = \frac{(\theta_c)^{\sigma-1} R P^{\sigma-1}}{\sigma} - (f_p)$$

$$\pi_2^D = \frac{(\theta_2)^{\sigma-1} R P^{\sigma-1}}{\sigma} - (f_p)$$

$$\pi_1^X = \frac{\tau^{1-\sigma}(\theta_c)^{\sigma-1} R P^{\sigma-1}}{\sigma} - (f_x)$$

$$\pi_2^X = \frac{\tau^{1-\sigma}(\theta_2)^{\sigma-1} R P^{\sigma-1}}{\sigma} - (f_x)$$

Also define $\Pi_i^D = \int_0^\infty \pi_i^D e^{-(1-\delta)t} dt - S_d$ and $\Pi_i^X = \int_0^\infty \pi_i^X e^{-(1-\delta)t} dt - S_x$ as the discounted net value of profits. The subscripts indicate the variety and the superscripts indicate the market the firms serves.

<u>Case 1:</u>

θ_x shows the cut-off for entry into export market. It is the productivity level below which it is not profitable for the firm to export. This cut-off is independent of the asset level of the firm. Note that because of the sunk cost of exporting, the cut-off is slightly different from Melitz (2003). θ_x^* and $\theta_{x,m}^*$ are defined as in section 2.1:

$$\theta_x^* = (\frac{S_x \delta}{f_x} + 1)^{\frac{1}{\sigma-1}} \theta_{x,m}^*$$

Is the productivity cut-off below which firms will not export.

<u>Case 2:</u>

Case 2 occurs when profits from selling variety 2 in the domestic market is higher than exporting good 1.

$$\pi_2^D \geq \pi_1^X$$

$$[(\frac{\theta_2}{\theta^*}^{\sigma-1}) - 1] f_p \geq n_x [(\frac{\theta_2}{\theta_{x,m}^*}^{\sigma-1}) - 1] f_x$$

Where $\theta_{x,m}^* = \tau(\frac{f_x}{f_p})^{\frac{1}{\sigma-1}} \theta^*$. Substituting for $\theta_{x,m}^*$ and noting $\theta_2 = c\theta_c$ we get the cut-off θ_{high} for which firms always prefer to add a new domestic product before starting to export. Substituting in gives cut-off θ_{high} as:

$$\theta_{high} = (\frac{f_x - f_p}{f_p(\frac{n_x}{\tau^{\sigma-1}} - c^{\sigma-1})})^{\frac{1}{\sigma-1}}\theta^*$$

Note that this condition requires $\frac{n_x}{\tau} \geq c$. Otherwise the value of expanding in the domestic market will always generate higher profits.

Case 3:

This case in a two product world shows situations in which the productivity level of the firm is such that an additional good in the domestic market and exporting are both profitable. Given the productivity of the firm, however, the profits generated by exporting the most productive good are higher than those from adding the second good in the domestic market:

$$\Pi_1^X \geq \Pi_2^D$$

Since the firm is credit constrained $a_0 < a_u$ the option to export *and* expand is not available immediately. The firm can take two different paths: 1) Introduce good 2 in the domestic market then export. 2) Export good 1 and then introduce good 2 in the domestic market. Profits generated from path 1 (therefore from production of good 2 in the domestic market) are smaller compared to the profits of exporting, but are generated earlier as the sunk cost of expanding in the domestic market is smaller than the sunk cost of exporting. To show the existence of an asset cut-off affecting the firm's sequence of product introduction, consider the present discounted value of paths described above. Once the firm becomes unconstrained it starts exporting good 2. Therefore, the path that gets the firm to the asset level which allows it to pay for the sunk cost of exporting good 2 must have a higher present value. Since this term exists in the present value of both paths, it can be ignored from the calculations without affecting the final result. Similarly, profits from selling good 1 in the domestic market are common among both paths and so it can be excluded form the PV in both cases. Below I only include the parts that are important for the sake of comparison.

$$PV(1) = \Pi_2^D e^{-(1-\delta)t_1} + \Pi_1^X e^{-(1-\delta)(t_1+t_2)}$$

$$PV(2) = \Pi_1^X e^{-(1-\delta)t_4} + \Pi_2^D e^{-(1-\delta)(t_4+t_5)}$$

Where:

$$t_1 = max\left\{0, \frac{S_d + 2f_p - a_0}{\pi_1^D}\right\}, t_2 = \begin{cases} \frac{S_X+2f+f_x-a_0}{\pi_D^1+\pi_2^D} & if\ t_1 = 0 \\ \frac{S_X+2f+f_x}{\pi_D^1+\pi_2^D} & ow \end{cases}$$

$$t_4 = max\left\{0, \frac{S_X + f_p + f_x - a_0}{\pi_1^D}\right\}, t_5 = \begin{cases} \frac{S_d+2f+f_x-a_0}{\pi_1^D+\pi_1^X} & if\ t_1 = 0 \\ \frac{S_d+2f+f_x}{\pi_1^D+\pi_1^X} & ow \end{cases}$$

First suppose $t_1 \neq 0$ and $t_4 \neq 0$. Then:

$$PV(1) = \Pi_2^D e^{-(1-\delta)\frac{S_d+2f_p-a_0}{\pi_1^D}} + \Pi_1^X e^{-(1-\delta)(\frac{S_d+2f_p-a_0}{\pi_1^D} + \frac{S_X+2f_p+f_x}{\pi_D^1+\pi_2^D})}$$

$$PV(2) = \Pi_1^X e^{-(1-\delta)\frac{S_X+f_p+f_x-a_0}{\pi_1^D}} + \Pi_2^D e^{-(1-\delta)(\frac{S_X+f_p+f_x-a_0}{\pi_1^D} + \frac{S_d+2f_p+f_x}{\pi_1^D+\pi_1^X})}$$

The equation capturing the firm being indifferent between the two path can be written as:

$$PV(1) = PV(2) \tag{35}$$

Note that as a_0 decreases t_4 increases much above t_1 (assuming $S_x > S_d$). The value of PV(2) decreases relative to PV(1) and therefore the first path is preferred. Similarly as a_0 increases t_4 and t_1 both decrease. As a_0 reaches S_x, the value of t_4 and t_1 get closer to each other. However, $\Pi_1^X > \Pi_2^D$, which means $PV(2) > PV(1)$. Now to show the uniqueness of such a_0 above (below) which path 2 (1) is preferred, it suffices to show that both PV(1) and PV(2) are monotonic -increasing- in a_0 and second derivative does not change signs:

$$\frac{\partial PV(1)}{\partial a_0} = \frac{1}{\pi_1^D}\Pi_2^D(1-\delta)e^{-(1-\delta)t_1} + \frac{1}{\pi_1^D}\Pi_1^X(1-\delta)e^{-(1-\delta)(t_1+t_2)} > 0$$

It can be seen that all terms in the FOC are positive and therefore the FOC is increasing in a_0. And the second derivative:

$$\frac{\partial^2 PV(1)}{\partial a_0^2} = (\frac{1}{\pi_1^D})^2 \Pi_2^D(1-\delta)^2 e^{-(1-\delta)t_1} + (\frac{1}{\pi_1^D})^2 \Pi_1^X(1-\delta)^{t_1+t_2} e^{-(1-\delta)(t_1+t_2)} > 0$$

This can similarly be shown for $PV(2)$ and other cases regarding t_1 and t_4. Also, because of the curvature of the present value functions, there will be another intersection between the two functions which would realise as a_0 reaches a_u. Note that only if $t_4 = 0$ since the profits from exporting are higher than profits from adding good 2 in the domestic market, an ordering dependent on the asset level will not exist.

Finally, the above result for the two product world can be generalised an n-product setting with similar reasoning. For any additional product that the firm wants to introduce the value of two paths should be compared. What is of interest here, is the point at which the firm becomes an exporter. Since export will always start with the most productive good, assuming the firm is already producing i varieties for the domestic market, the paths that should be compared are 1) Introducing I_1 or 2) Introducing D_{i+1} . Therefore, first the comparison is between I_1 and D_2 which is the similar to before. The second round, if D_2 is chosen, is between I_1 and D_3. As the profit of D_2 is common in both present value functions, with the same reasoning as in Case 3, it can be excluded from the PV expression. This additional profit will only be present in the exponents of discount factor which is an indicator of the time it would take for the firm to introduce that good. The rest of the proof is exactly similar to the two-good world.

A.4. Proof of Proposition 2

A liquidity constrained firm has an optimal ordering of introducing new products that follows. In a 2 product world this would be:

Path 1: Exporting variety 1 then adding variety 2 in the domestic market.

Path 2: Add variety 2 to the domestic market and then exporting variety 1.

In both cases exporting variety 2 comes at the final stage since the profits from exporting 1 are higher and we have imposed the selection into exporting condition. The path the firm follows depends on the productivity level as a function of the initial wealth and has been already discussed in detail in proposition 1. The value of productivity cut-off θ_{prop} determining the path the firm follows is the unique solution to equation 35.

The value of entry is composed of 2 different parts:

1) The value of following path 1.

2) The value of following path 2.

Each part is weighted appropriately by the respective probability. Since the expected profit from domestic production of variety 1 is a common term it is written in the beginning (line 2). Lines 3 and 4 show the value of entry for productivity levels that follow path 2. Lines 5 and 6 are the value of entry for productivity levels following path 1.

$$v^e(a_0) = \tag{1}$$

$$\bar{\Pi}_1^D P(\theta^* \leq \theta_c) \tag{2}$$

$$+ \bar{\Pi}_2^D P(\frac{\theta^{**}}{c} \leq \theta_c < \theta_{prop}) e^{-(1-\delta)\bar{t_1}} \tag{3}$$

$$+ \bar{\Pi}_1^X P(\theta_x^* \leq \theta_c < \theta_{prop}) e^{-(1-\delta)(\bar{t_1}+\bar{t_3})} \tag{4}$$

$$+ \bar{\Pi}_2^D P(max\{\frac{\theta^{**}}{c}, \theta_{prop}\}) \leq \theta_c) e^{-(1-\delta)\bar{t_2}+\bar{t_4}} \tag{5}$$

$$+ \bar{\Pi}_1^X P(max\{\theta_x^*, \theta_{prop}\} \leq \theta_1) e^{-(1-\delta)\bar{t_4}} \tag{6}$$

$$+ \bar{\Pi}_2^X P(\frac{\theta_x^*}{c} \leq \theta_1) e^{-(1-\delta)min\{\bar{t_3}+\bar{t_1}, \bar{t_2}+\bar{t_4}\}} \tag{7}$$

To be more explicit about how each term in the value of entry is written, take the expression in line 3. This shows the expected present discounted value of profits from production of variety 2 to sell in the domestic market. $\bar{\pi}_{D_2}$ is expected domestic profits of variety 2 conditional un successful entry and finding path 2 optimal. $P(\frac{\theta^{**}}{c} \leq \theta_c)$ is the probability that the firm will have sufficiently high productivity for variety 2 such that the net present value of profits for this variety are positive. $P(\theta_c < \theta_{prop})$ is the probability of having a productivity level such that it will be optimal for the firm to follow path 2. Recall from proposition x that θ_{prop} is the productivity cut-off associated with asset level a_0 below which firms prefer to expand in the domestic market first. Finally, $e^{-(1-\delta)(t+\bar{t_1})}$ is the discount factor. $\bar{t_1}$ shows the expected time at which the firm can generate profits from selling variety 2 in the domestic market. Other terms in the value of entry are defined similarly.

Profits can then be expressed as a function of the productivity level and t_1 to t_4 can be expressed as the known parameters of the model. θ_{prop} is the unique solution to equation 35. Substituting for all these values, we can see that the unknowns in the value of entry are θ^* and θ_x^*. Finally, as in Melitz (2003), the relationship between the productivity cut-offs is used to express $v^e(a_0)$ as a function of θ^*.

Define:

$$
\begin{aligned}
t_1 &= max\left\{\frac{S_d + 2f_p - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p}, 0\right\} \\
t_2 &= \frac{S_d + 2f_p + f_X - (a_0 - S_x)}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p + ((\frac{\theta}{\theta^*_{x,m}})^{\sigma-1} - 1)f_x} \\
t_3 &= \frac{S_d + 2f_p - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p + ((\frac{c\theta}{\theta^*})^{\sigma-1} - 1)f_p} \\
t_4 &= max\left\{\frac{S_x + f_p + f_x - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p + ((\frac{\theta}{\theta^*_{x,m}})^{\sigma-1} - 1)f_x}, 0\right\}
\end{aligned}
$$

Now define θ^{**} as the productivity cut-off above which net present value of profits from variety 2 are positive and as in section 1.5.1:

$$
\theta^{**} = (\frac{S_d\Delta}{f_p} + 1)^{\frac{1}{\sigma-1}}\theta^*
$$

Given the above definitions, the value of entry can be written as:

$$
\begin{aligned}
&v^e(a_0) = \\
&\quad \int_0^\infty \int_{\theta^*}^\infty e^{-(1-\delta)t} f_p((\frac{\theta}{\theta^*})^{\sigma-1} - 1)g(\theta)d\theta dt \\
&\quad + \int_0^\infty \int_{\frac{\theta^{**}}{c}}^{\theta_{prop}} e^{-(1-\delta)t}(f_p((\frac{c\theta}{\theta^*})^{\sigma-1} - 1) - S_d)e^{-(1-\delta)t_1} g(\theta)d\theta dt \ \mathbf{1}(\theta_{prop} > \frac{\theta^{**}}{c}) \\
&\quad + \int_0^\infty \int_{\theta^*_x}^{\theta_{prop}} (e^{-(1-\delta)t} f_x((\frac{\theta}{\theta^*_x})^{\sigma-1} - 1) - S_x)e^{-(1-\delta)(t_1+t_3)} g(\theta)d\theta dt \ \mathbf{1}(\theta_{prop} > \theta^*_x) \\
&\quad + \int_0^\infty \int_{max\{\frac{\theta^{**}}{c}, \theta_{prop}\}}^\infty (e^{-(1-\delta)t} f_p((\frac{c\theta}{\theta^*})^{\sigma-1} - 1) - S_d)e^{-(1-\delta)(t_2+t_4)} g(\theta)d\theta dt \\
&\quad + \int_0^\infty \int_{max\{\theta^*_x, \theta_{prop}\}}^\infty (e^{-(1-\delta)t} f_x((\frac{\theta}{\theta^*_x})^{\sigma-1} - 1) - S_x)e^{-(1-\delta)t_4} g(\theta)d\theta dt \\
&\quad + \int_0^\infty \int_{\frac{\theta^*_x}{c}}^\infty (e^{-(1-\delta)t} f_x((\frac{c\theta}{\theta^*_x})^{\sigma-1} - 1) - S_x)e^{-(1-\delta)min\{t_3+t_1, t_2+t_4\}} g(\theta)d\theta dt
\end{aligned}
$$

To show continuity note that the sum of a finite number of continuous functions is continuous. Therefore, expression for value of entry can be broken down to a finite sum. Also, product of a finite number of continuous functions is a continuous function. Given above, it is sufficient to show that each integral term in the value of entry is continuous. $g(\theta)$ is a probability distribution function and therefore is continuous. $(\frac{c\theta}{\theta^*})^{\sigma-1} - 1) - S_d$ (or the equivalent for exporting) is continuous as long as $\theta^* \neq 0$. Finally, the continuity of the discount should be checked. Taking the second term in the expression for value of entry, it must be shown that $e^{-(1-\delta)t_1}$ is continuous given the bounds. t_1 is defined with a max operator, so it can be written as:

$$t_1 = 0.5(\frac{S_d + 2f_p - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p} + |\frac{S_d + 2f_p - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p}|)$$

For $a_0 < S_d + 2f_p$ the fraction defined above is continuous as long as $\theta \neq \theta^*$. Now, note that the bounds on the integral imply that $\theta > \theta^*$. For $a_0 > S_d + 2f_p$ the value is equal to zero. Therefore, the limit should be checked:

$$\lim_{a_0 \to S_d + 2f_p} (\frac{S_d + 2f_p - a_0}{((\frac{\theta}{\theta^*})^{\sigma-1} - 1)f_p}) = 0$$

Continuity can similarly be shown for other terms in the expression.
To show the function is increasing in a_0, note that with an increase in a_0, the previous set of production choices remain available to the firm. Therefore $v^e(a_0)$ will not decrease. However further increasing the initial asset, decreases the delay in entering new markets and increases the value of entry. However, for firms that are not financially constrained, increasing the asset level has no effect on value of entry.

Appendix B: Calibration

B.1. Numerical Method

In order to obtain the solution to the problem a number of firms are simulated. Each firm has three attributes: productivity level, initial asset level and initial cost shock. Productivity level has a grid on domain [9, 12.5] to match Mayer et al. (2014). The value of asset is between f_p and $\bar{a}$, where $\bar{a}$ is high enough such that firms are not financially constrained. The cost shock is a two state symmetric Markov process and its value is chosen such that it is never the main driver of the profits. The initial draw for the productivity level is from an exponential distribution. The draw for asset level is a uniform distribution and the initial cost shock can take either one of the two values with equal probability.

To solve, first, I make an initial guess for the aggregate price level P. Using the guess, the problem of the firm is solved, value functions are calculated with backward induction and the policy functions are derived. Then the free entry condition is applied to determine the active firms. Given the above, the stationary distribution of firms is calculated and the aggregate price P is updated. The procedure is repeated until the aggregate price converges to its equilibrium level.

B.2. Discrete time set up

Firm's problem:

$$V^X(\theta_c, c, a_t, \epsilon_t) = \max_{p,q,X_i,D_i} E_t \sum_i J_{i,t}\pi_t^D(\theta_i) - \sum_i D_{i,t}S_d +$$
$$\sum_i J_{i,t}^X \pi_t^X(\theta_i) - \sum_i X_{i,t}S_x + (1-\delta)E_t\big[V_{t+1}^X(\theta_c, c, a_{t+1}, \epsilon_{t+1})\big]$$

Subject to:

$$J_{i,t} = max\{J_{i,t-1}, D_{i,t}\}$$
$$J_{i,t}^X = max\{J_{i,t-1}^X, X_{i,t}\}$$
$$\theta_i = c^{i-1}\theta_c \qquad i \in N$$
$$a_t \geq \textstyle\sum_i J_{i,t}f_p + \sum_i D_{i,t}S_d + \sum_{J^X \in J} J_{i,t}^X f_x + \sum_i D_{i,t}S_d$$
$$a_t = r(a_{t-1} - \textstyle\sum_i D_{i,t-1}S_d - \sum_i X_{i,t-1}S_x) + \sum_i J_i\pi^D(\theta_i)\sum_i J_i^X\pi^X(\theta_i)$$

Where r is the interest rate and now there is an ϵ shock to the profits of the firm. The other terms are defined similar to the setting described in previous section.

Figure 2 corresponds to dynamics that were not present in a continuous time framework. In discrete time the uniqueness result is not guaranteed. As an example, it is possible that firms with a very high level of productivity become unconstrained after one period and enter all the profitable markets.